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Tangent Similarity Measures of Picture Fuzzy Sets and Their Applications to Pattern Recognition

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ABSTRACT: Picture fuzzy set (PFS), is a newly developed apparatus to treaty with uncertainties in problems where the opinions are yes, no, neutral, and refusal types. It can work very efficiently in many practical problems, such as pattern recognition, artificial intelligence, robotic, expert and knowledge-based systems etc. The similarity measure of PFSs is an imperative appliance to adopt the involving of two PFSs. It is applied in varied twigs of our daily life complications, where decision-making, pattern recognition, medical diagnosis, etc. are remarkable. In this article, the conception of tangent similarity measures of PFSs is introduced and designated some of their related properties. Then the tangent similarity measures of PFSs are applied to pattern recognition.Finally, to show the efficiency and the validity of our proposed tangent similarity measures, a comparative study with the existing methods is illustrated.

Keywords: Picture fuzzy set; Tangent similarity measure; Weighted tangent similarity measure; Pattern recognition

1. INTRODUCTION

Picture fuzzy set (PFS), was first introduced by Cuong and Kreinovich [1, 2] which is a direct extension of the fuzzy set (FS) [3] and the intuitionistic fuzzy set (IFS) [4] by including the idea of positive, negative, and neutral membership degree. The PFS also describe the refusal degree of each element of a universal set. The concept of the PFS is more suitable in situations where the opinions are yes, no, neutral, and refusal types. The similarity measures of the PFSs have gained much attention to the researchers due to its successful applications in different fields of science and engineering. Wei [5] established the similarity measure between PFSs based on the cosine functions and applied it in strategic decision making. After that, Wei [6] also developed the some similarity measures for PFSs and applied them in building material recognition and minerals field recognition. Dice similarity measure of PFSs is proposed by Joshi [7] and applied it in multi-criteria decision making problems. The generalized Dice similarity measure between picture fuzzy sets is also

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established by Wei and Gao [8] and applied it to building material recognition. A similarity measure of the PFSs based on entropy is introduced by Thao [9] and applied for MCDM solving the supplier problems. Bi-parametric similarity and distance measures are described by Khan et al. [10] and applied it in medical diagnosis problem. Luo [11] proposed a similarity measure based on the constituent functions of a picture fuzzy set, and applied it to pattern recognition. Luo [12] also developed similarity measure between picture fuzzy sets based on relationship matrix and applied to multiple-attribute decision making. Similarity measures of PFSs are also studied by (see [13-15]).

In this article, tangent similarity measures for PFSs are proposed and describe some related properties of them. The application of tangent similarity measures of PFSs to pattern recognition problem is also described. Finally, the comparative analysis of our proposed methods with existing methods is presented.

The main contributions of this study are described below:

• We suggest a new PF similarities measures that can describe positive, neutral and negative membership degrees of any elements between two PFSs.
• We demonstrate its use in pattern recognition.

The remainder of the paper is structured as follows: In section 2 some preliminary definitions are described which are essential to rest of the paper. In section 3, the definitions of tangent similarity measure and weighted tangent similarity measure for PFSs are given and investigated some related properties of them. In section 4, an application of proposed tangent similarities measures to pattern recognition is illustrated. In section 5, the comparative analysis of our proposed methods with existing methods is specified.

2. PRELIMINARIES

Definition 2.1: [3] Let X be non-empty set. A fuzzy set A in X is given by

$$A = \{(x, \mu_A(x)) : x \in X\}, \text{ where } \mu_A : X \to [0,1].$$

Definition 2.2: [4] An IFS A in X is given by

$$A = (x, \mu_A(x), \nu_A(x) : x \in X),$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1; \forall x \in X$.

The values $\mu_A(x)$ and $\nu_A(x)$ represent respectively the membership degree and non-membership degree of the element to the set A.

Definition 2.3: [1, 2] A picture fuzzy set (PFS) A on a universal set X is of the form

$$A = (x, \mu_A(x), \eta_A(x), \nu_A(x) : x \in X),$$

where $\mu_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\eta_A(x) \in [0,1]$ is called the degree of neutral membership of x in A and $\nu_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\mu_A(x), \eta_A(x)$ and $\nu_A(x)$ satisfies the following condition:

$$0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1; \forall x \in X.$$

Here $1 - \mu_A(x) + \eta_A(x) + \nu_A(x); \forall x \in X$is called the degree of refusal membership of x in A.

Definition 2.4: [1, 2] Let $A = (x, \mu_A(x), \eta_A(x), \nu_A(x): x \in X)$ and

$B = (x, \mu_B(x), \eta_B(x), \nu_B(x): x \in X)$ be two picture fuzzy sets in the universal set X. Then the subset is defined as follows:

$$A \subseteq B \text{ iff } \forall x \in X, \mu_A(x) \leq \mu_B(x), \eta_A(x) \leq \eta_B(x) \text{ and } \nu_A(x) \geq \nu_B(x).$$

3. TANGENT SIMILARITY MEASURES FOR PICTURE FUZZY SETS

Definition 3.1: Let $A = (x, \mu_A(x), \eta_A(x), \nu_A(x): x \in X)$ and

$B = (x, \mu_B(x), \eta_B(x), \nu_B(x): x \in X)$ be two picture fuzzy sets in the universal set X. Then the tangent similarity measure between A and B is defined as:

$$T(A,B) = 1 - \frac{1}{n}\left(\sum_i^n \tan\frac{\pi}{12}(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)\right) \quad (1)$$

Theorem 3.2: Let A, B and C be three picture fuzzy sets (PFSs) on X. Then,

1. $0 \leq T(A,B) \leq 1$,
2. $T(A,B) = T(B,A)$,
3. $T(A,B) = 1$ if and only if $A = B$,
4. if $A \subseteq B \subseteq C$, then $T(A,B) \geq T(A,C)$ and $T(B,C) \geq T(A,C)$.

Proof: 1.

It is discerned that, the tangent function is monotonic increasing in the interval $[0, \frac{\pi}{4}]$.

It is also lies in the interval $[0, 1]$.

Therefore, we have $0 \leq T(A,B) \leq 1$.

2. From the definition 3.1, we have

$$T(A,B) = 1 - \frac{1}{n}\left(\sum_i^n \tan\frac{\pi}{12}(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)\right)$$

$$= 1 - \frac{1}{n}\left(\sum_i^n \tan\frac{\pi}{12}(|\mu_B(x_i) - \mu_A(x_i)| + |\eta_B(x_i) - \eta_A(x_i)| + |\nu_B(x_i) - \nu_A(x_i)|)\right)$$

$$= T(B, A).$$

Therefore, $T(A, B) = T(B, A)$.

3. Let $A = B$, then we have $\mu_A(x_i) = \mu_B(x_i)$, $\eta_A(x_i) = \eta_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$.

This implies that $|\mu_A(x_i) - \mu_B(x_i)| = 0$, $|\eta_A(x_i) - \eta_B(x_i)|$ and $|\nu_A(x_i) - \nu_B(x_i)| = 0$.

Therefore, $T(A, B) = 1 - \frac{1}{n}(\sum_i^n \tan(0)) = 1$.

Conversely, suppose that $T(A, B) = 1$.

This implies that $|\mu_A(x_i) - \mu_B(x_i)| = 0$, $|\eta_A(x_i) - \eta_B(x_i)|$ and $|\nu_A(x_i) - \nu_B(x_i)| = 0$.

Hence $\mu_A(x_i) = \mu_B(x_i)$, $\eta_A(x_i) = \eta_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$.

Therefore, $A = B$.

4. Since $A \subseteq B \subseteq C$, we have

$\mu_A(x_i) \le \mu_B(x_i), \eta_A(x_i) \le \eta_B(x_i)$, $\nu_A(x_i) \ge \nu_B(x_i)$, $\mu_B(x_i) \le \mu_C(x_i), \eta_B(x_i) \le \eta_C(x_i)$, $\nu_B(x_i) \ge \nu_C(x_i)$ and $\mu_A(x_i) \le \mu_C(x_i), \eta_A(x_i) \le \eta_C(x_i)$, $\nu_A(x_i) \ge \nu_C(x_i)$.

Therefore,

$|\ \mu_A(x_i) - \mu_B(x_i)| \le |\ \mu_A(x_i) - \mu_C(x_i)|$,$|\ \eta_A(x_i) - \eta_B(x_i)| \le |\ \eta_A(x_i) - \eta_C(x_i)|$ and $|\ \nu_A(x_i) - \nu_B(x_i)| \le |\ \nu_A(x_i) - \nu_C(x_i)|$.

Now,

$$T(A, B) = 1 - \frac{1}{n}\left(\sum_i^n \tan\frac{\pi}{12}(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)\right)$$

$$\ge 1 - \frac{1}{n}\left(\sum_i^n \tan\frac{\pi}{12}(|\mu_A(x_i) - \mu_C(x_i)| + |\eta_A(x_i) - \eta_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)|)\right)$$

$$= T(A, C)$$

This implies, $T(A, B) \ge T(A, C)$.

Similarly, we can prove that $T(B, C) \ge T(A, C)$.

Therefore, we have finished the proofs.

Definition 3.3: Let $A = (x, \mu_A(x), \eta_A(x), \nu_A(x): x \in X)$ and

$B = (x, \mu_B(x), \eta_B(x), \nu_B(x): x \in X)$ be two picture fuzzy sets in the universal set X. Then the weighted tangent similarity measure between A and B is defined as:

$$WT(A,B) = 1 - \frac{1}{n} w_i \left(\sum_i^n \tan \frac{\pi}{12} (|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \right) \quad (2)$$

where $\sum_{i=1}^n w_i = 1$.

Theorem 3.4: Let A, B and C be three picture fuzzy sets (PFSs) on X. Then,

1. $0 \le WT(A,B) \le 1$,
2. $WT(A,B) = WT(B,A)$,
3. $WT(A,B) = 1$if and only if $A = B$,
4. if $A \subseteq B \subseteq C$, then $WT(A,B) \ge WT(A,C)$ and $WT(B,C) \ge WT(A,C)$.

Proof: Trivial.

4. APPLICATIONS OF THE TANGENT SIMILARITY MEASURES TO PATTERN RECOGNITION

Suppose that there m patterns which are represented by PFSs

$A_j = \{(x, \mu_{A_j}(x), \eta_{A_j}(x), \nu_{A_j}(x): x \in X\} \in PFSs(X) (j = 1, 2, 3 \cdots \cdots m)$. Assume that there is a sample pattern to be recognized is represented by a PFS $B = \{(x, \mu_B(x), \eta_B(x), \nu_B(x): x \in X\} \in PFSs(X)$.

$$\text{Set } T(A_j, B) = \max_{1 \le j \le m} \{T(A_j, B)\} \quad (3)$$

Aoccording to the principle of maximum degree, we can conclude that sample B belong to the pattern A_j.

Example 4.1: Let four patterns represented by PFSs in $X = \{x_1, x_2, x_3\}$ as

$$A_1 = \{(0.73, 0.16, 0.04), (0.60, 0.33, 0.02), (0.85, 0.06, 0.08)\},$$
$$A_2 = \{(0.89, 0.08, 0.03), (0.16, 0.56\ 0.21), (0.74, 0.16, 0.10)\},$$
$$A_3 = \{(0.33, 0.51, 0.12), (0.54, 0.31, 0.15), (0.16, 0.71, 0.05)\},$$
$$A_4 = \{(0.17, 0.55, 0.14), (0.1, 0.00, 0.00), (0.91, 0.00, 0.05)\}.$$

Consider a sample pattern $B \in PFSs(X)$ which will be reconized, where

$B = \{(0.75, 0.05, 0.14), (0.81, 0.03, 0.10), (0.55, 0.32, 0.05)\}$.

By applying the Eq. (1), we have,

$$T(A_1, B) = 0.8741,$$
$$T(A_2, B) = 0.8240,$$

$$T(A_3, B) = 0.7964,$$
$$T(A_4, B) = 0.7678.$$

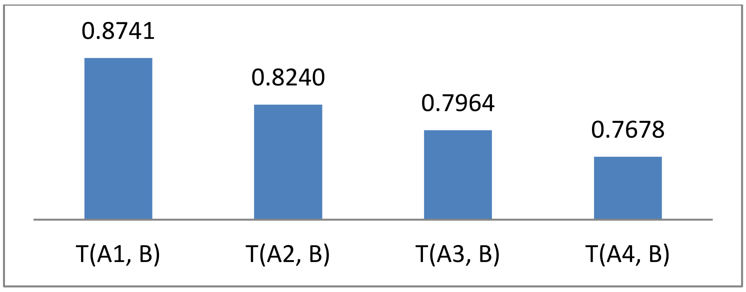

Figure 1: The tangent similarity measures between B and A_i; $i = 1, 2, 3, 4$

From the above numerical results it is obvious that the degree of tangent similarity between B and A_1 is the largest one, the degree of tangent similarity between B and A_2 ranks the second, the degree of tangent similarity between B and A_3 ranks the third, the degree of tangent similarity between B and A_4 is the smallest one. Therefore, the pattern B should belong to the pattern A_1 according to the principle of the maximum degree of tangent similarity between PFSs.

Assume that the weight of $x_i (i = 1, 2, 3)$ is: $w = (0.55, 0.24, \; 0.15)^T$. By applying the Eq. (2), we have

$$WT(A_1, B) = 0.9681,$$
$$WT(A_2, B) = 0.9532,$$
$$WT(A_3, B) = 0.9325,$$
$$WT(A_4, B) = 0.9198.$$

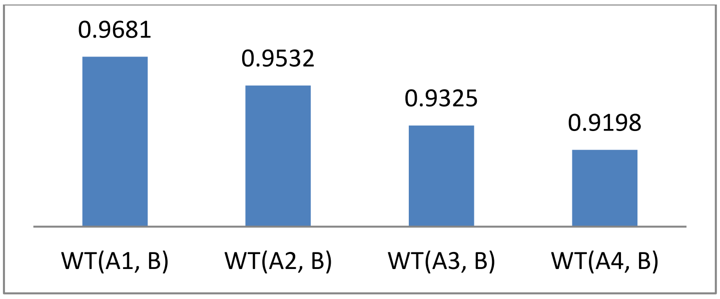

Figure 2: The weighted tangent similarity measures between B and A_i; $i = 1, 2, 3, 4$

In this case, the pattern B also should belong to the pattern A_1.

5. COMPARISON STUDIES

In this section, in order to show the effectivity of our proposed methods, we shall compare the proposed methods with the methods of cosine similarity of picture fuzzy sets which were proposed by Wei [13].

By using the cosine similarity measure of picture fuzzy, we can calculate the similarity $S(A_j, B)$ between A_j $(j = 1; 2; 3; 4)$ and B by using equation (4) in Ref.[13]:

$$S(A_1, B) = 0.5179,$$
$$S(A_2, B) = 0.4365,$$
$$S(A_3, B) = 0.3575,$$
$$S(A_4, B) = 0.2537.$$

The greater the value of $S(A_j, B)$ is, the alternative is closer B to A_j. In this case, B should belong to A_1.

Again, by using the weighted cosine similarity measure of picture fuzzy, we also can compute the similarity $WS(A_j, B)$ between A_j $(j = 1; 2; 3; 4)$ and B by using equation (5) in Ref.[13]:

$$WS(A_1, B) = 0.5030,$$
$$WS(A_2, B) = 0.4822,$$
$$WS(A_3, B) = 0.3190,$$
$$WS(A_4, B) = 0.1917.$$

The greater the value of $S(A_j, B)$ is, the alternative is closer B to A_j. In this case, B also should belong to A_1.

From the above analysis, it can be seen that the proposed methods is effective.

6. CONCLUSIONS

The similarity measure of picture fuzzy set (PFS) becomes an important topic in applications of PFSs to our real life problems. A host of researchers developed various similarity measures of PFSs and applied them in different branches of science and engineering. In this article, the concepts of tangent similarity and weighted tangent similarity measures of PFSs have been introduced and described some structural properties of them. The proposed similarities measures are also have been applied to pattern recognition problem. Finally, a comparison study of our proposed methods with existing methods is discussed to show the affectivity and consistence of our proposed methods.

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