UITS Journal of Science & Engineering ❖ **Volume: 7, Issue: 1**
ISSN: 2521-8107, January-2020

Numerical Solution of Fuzzy Non-linear Equation Using Modified Bisection Method

Md. Yasin Ali[1], Ismat Ara Khan[2]

ABSTRACT: In this paper, new arithmetic operations on triangular fuzzy numbers are introduced. Then based on these operations, the classical Bisection method is modified for solving fuzzy non-linear equation over triangular fuzzy numbers. A numerical example is also given to show the applicability of the method. Finally, graphical depiction of the roots has also been drawn so that anyone can achieve the idea of converging to the root.

Key words: Triangular fuzzy number, Fuzzy non-linear equation, Fuzzy arithmetic operations, Ranking number, Modified Bisection method.

1. INTRODUCTION

One area of fuzzy set theory in which fuzzy numbers and arithmetic operations on fuzzy numbers play a fundamental role is fuzzy equations. These equations in which coefficients and unknowns are fuzzy numbers and formulae are constructed by operations of fuzzy arithmetic. Fuzzy non-linear equation plays a major role in various areas such as applied mathematics, engineering, and social sciences. Many researchers have studied on the solution of fuzzy non-linear equations. Subhash and Sathya [1] proposed a method using linear interpolation to solve a non-linear equation $f(x)=0$, which is a modification of fuzzy Newton-Raphson method. Gautam and Shirin [2, 3, 4] proposed methods to solve a fuzzy non-linear equation with the help Modified Fixed Point Algorithm, Bisection Algorithm and Secant Algorithm. Abbasbandy and Asady [5] have considered Newton's method for solving fuzzy non-linear equations. Ali et al. [6] used modified secant method to solve no-linear fuzzy equation. In this paper, we have introduced new arithmetic operations on triangular fuzzy numbers. Then, classical Bisection method is modified and used to solve fuzzy non-linear equation by using these new arithmetic operations on triangular fuzzy numbers.

1 Assistant Professor, Department of Electrical and Electronic Engineering, UITS.
* Corresponding author: Email: ali.mdyasin56@gmail.com
2 Lecturer, Department of Electrical and Electronic Engineering, UITS.
Email: iakhan06@hotmail.com

The paper is organized as follows. In section 2 we have discussed about basic definitions and notations which are used throughout this paper. In section 3, we have given arithmetic operations on triangular fuzzy number. In section 4, Classical Bisection method is modified by using these new operations. In section 5, the precise solution of the non-linear fuzzy equation is obtained with the help of the proposed method.

2. PRELIMINARIES

Definition 2.1: [7] A fuzzy set $\tilde{A}$ is defined by

$\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A the second element $\mu_A(x)$ belongs to the interval [0, 1], called membership function.

Definition 2.2: [7] If a fuzzy set is convex and normalized, and its membership function is defined in $\Re$ and piecewise continuous, it is called as fuzzy number. So a fuzzy numbers represent a real number interval whose boundary is fuzzy. Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number $\Re$. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Generally a fuzzy interval is represented by two end points a_1, a_3 and a peak point a_2 as [a_1, a_2, a_3] (Figure-1)

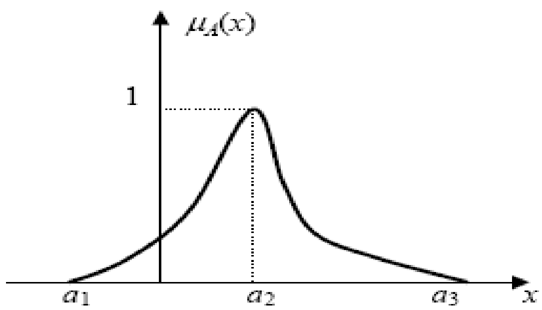


Figure 1: Fuzzy Number.

Definition 2.3: [7] A fuzzy number $\tilde{A}$ on $\Re$ is said to be a triangular fuzzy number (TFN) or linear real fuzzy number (LRFN) if its membership function $\mu : \Re \to [0,1]$ has the following characteristics:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, x < a \\ \dfrac{x-a}{b-a}, a \le x \le b \\ \dfrac{c-x}{c-b}, b \le x \le c \\ 0, x > c \end{cases}$$

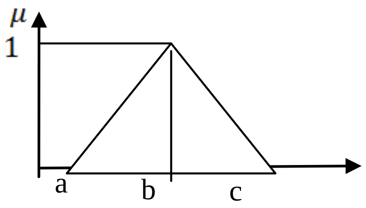


Figure 2 : Triangular fuzzy number.

The triangular fuzzy number is denoted by $\tilde{A} = (a,b,c)$. Any real number r can also be written as triangular fuzzy number as $\tilde{A} = (r,r,r)$.

Definition 2.4: [3] A non-linear equation over triangular fuzzy number is called a fuzzy non-linear equation. The equation of the form $\tilde{f}(\tilde{x}) = \tilde{0}$ is called fuzzy non-linear equation, where $\tilde{f} : TF(\Re) \to TF(\Re)$ is a non-linear function. For example $\tilde{x}^3 + \tilde{x}^2 - \tilde{1} = \tilde{0}$ is a fuzzy non-linear equation.

3. ARITHMETIC OPERATIONS ON TRIANGULAR FUZZY NUMBER

Ma et al. [8] first proposed a new arithmetic operation on triangular fuzzy number, based on location index number and fuzziness index functions. The location index number is taken in the ordinary arithmetic operations, whereas the fuzziness index functions are considered to be lattice rule. Here, we have proposed a new fuzzy arithmetic operations based upon mid index number and left and right fuzziness index numbers. The mid index number is taken in the ordinary arithmetic, while the left and right fuzziness index numbers are considered as max and min composition.

Definition 3.1: For an arbitrary triangular fuzzy number $\tilde{A} = (a,b,c)$, the number $a_0 = \left(\dfrac{a+c}{2}\right)$ is said to be mid index number $\tilde{A}$ and $a^L = a_0 - a$ and $a^R = c - a_0$ is said to be left and right fuzziness index numbers of $\tilde{A}$ respectively. Hence every triangular fuzzy number $\tilde{A} = (a,b,c)$ can also be represented by $\tilde{A}^* = (a_0, a^L, a^R)$.

Definition 3.2: For arbitrary triangular fuzzy numbers $\tilde{A}^* = (a_0, a^L, a^R)$ and $\tilde{B}^* = (b_0, b^L, b^R)$, the arithmetic operations on them are defined by

(1) Addition: $\tilde{A}^* + \tilde{B}^* = (a_0, a^L, a^{*R}) + (b_0, b^L, b^R)$

$$= (a_0 + b_0, \max\{a^L, b^L\}, \max\{a^R, b^R\},$$

(2) Subtraction: $\tilde{A}^* - \tilde{B}^* = (a_0, a^L, a^R) + (b_0, b^L, b^R)$

$$= (a_0 - b_0, \min\{a^L, b^L\}, \min\{a^R, b^R\},$$

(3) Multiplication: $\tilde{A}^* \times \tilde{B}^* = (a_0, a^L, a^R) + (b_0, b^L, b^R)$

$$= (a_0 \times b_0, \max\{a^L, b^L\}, \max\{a^R, b^R\},$$

(4) Division: $\tilde{A}^* \div \tilde{B}^* = (a_0, a^L, a^R) + (b_0, b^L, b^R)$

$$= (a_0 \div b_0, \min\{a^L, b^L\}, \min\{a^R, b^R\},$$

(5) Scalar Multiplication: $k\tilde{A}^* = k(a_0, a^L, a^{*R})$, where k is a real number

$$= (ka_0, a^L, a^R),$$

(5) Scalar Division: $\tilde{A}^* / k = (a_0, a^L, a^{*R}) / k$, where k is a real number

$$= (a_0 / k, a^L, a^R).$$

Definition 3.3: Let $TF(\Re)$ denotes the set of all triangular fuzzy numbers. Let us define a ranking function $\Re : TF(\Re) \to \Re$ which maps all triangular fuzzy numbers into $\Re$. If $\tilde{A} = (a,b,c)$ is a triangular fuzzy number, then the defuzzify ranking number is given by,

$$\Re(\tilde{A}) = \frac{a+b+c}{3}$$

$$= \frac{a^R + 3a_0 - a_L}{3}$$

4. MODIFIED BISECTION METHOD TO SOLVE FUZZY NON-LINEAR EQUATION

Consider the fuzzy non-linear equation $\tilde{f}(\tilde{x}) = \tilde{0}$

Let $\tilde{x}_1 = (a_1, b_1, c_1)$ and $\tilde{x}_2 = (a_2, b_2, c_2)$ be two initial approximations.

If $\Re(\tilde{f}(\tilde{x}_1^*))$ and $\Re(\tilde{f}(\tilde{x}_2^*))$ opposite sign (say $\Re(\tilde{f}(\tilde{x}_1^*)) < 0$ and $\Re(\tilde{f}(\tilde{x}_2^*)) > 0$. Then the root lies between $\tilde{x}_1^*$ and $\tilde{x}_2^*$. Let it be $\tilde{x}_3^*$, and $\tilde{x}_3^* = \dfrac{\tilde{x}_1^* + \tilde{x}_2^*}{2}$. If $\Re(\tilde{f}(\tilde{x}_1^*))$ and $\Re(\tilde{f}(\tilde{x}_3^*))$ opposite sign, then there exists a root between $\tilde{x}_1^*$ and $\tilde{x}_3^*$. Let it be $\tilde{x}_4^*$, and $\tilde{x}_4^* = \dfrac{\tilde{x}_1^* + \tilde{x}_3^*}{2}$. On the other hand if $\Re(\tilde{f}(\tilde{x}_2^*))$ and $\Re(\tilde{f}(\tilde{x}_3^*))$ opposite sign then there exists a root between $\tilde{x}_2^*$ and $\tilde{x}_3^*$. Let it be $\tilde{x}_4^*$, and $\tilde{x}_4^* = \dfrac{\tilde{x}_2^* + \tilde{x}_3^*}{2}$.

Proceeding in this way, sequences of roots $\{\tilde{x}_n\}$ can be obtained and it converges to the exact root.

ALGORITHM

INPUT: Convert the initial approximation $\tilde{x}_1 = (a_1, b_1, c_1)$ and $\tilde{x}_2 = (a_2, b_2, c_2)$ into $\tilde{x}_1^* = (a_{1_0}, a_1^L, a_1^R)$ and $\tilde{x}_2^* = (a_{2_0}, a_1^L, a_1^R)$, tolerance TOL; maximum number of iterations n.

OUTPUT: approximation solution $\tilde{x}_n$ or message of failure.

Step-1: Set $i = 1$

$$fa = \tilde{f}(\tilde{x}_1^*)$$

Step-2: While $i \le n$ do steps 3-6.

Step-3: Set $\tilde{x}_n^* = \dfrac{\tilde{x}_{n-1}^* + \tilde{x}_{n-2}^*}{2}$ (compute $\tilde{x}_n^*$)

$$fp = \tilde{f}(\tilde{x}_n^*)$$

Step-4: If $fp = \tilde{0}$ or $\dfrac{\tilde{x}_n^* - \tilde{x}_{n-1}^*}{2} < TOL$ then

OUTPUT ($\tilde{x}_n^*$) and convert into $\tilde{x}_n$; (The procedure was successful)

STOP

Step-5: Set $i = i + 1$

Step-6: If $fa, fp > 0$ then set $\tilde{x}_1^* = \tilde{x}_n^*$ (Update $\tilde{x}_1^*$)

$fa = fp$

Else set $\tilde{x}_2^* = \tilde{x}_n^*$

Step-7: OUTPUT ("The method failure after n iteration, n" n);

(The procedure is unsuccessful.)

STOP.

5. NUMERICAL EXAMPLE

Solve the fuzzy non-linear equation
$(1,1,1)\tilde{x}^3 + (4,4,4)\tilde{x}^2 - (10,10,10) = (0,0,0)$

Solution: Suppose that $\tilde{f}(\tilde{x}) = (1,1,1)\tilde{x}^3 + (4,4,4)\tilde{x}^2 - (10,10,10)$ (5.1)

Let $\tilde{x}_1 = (0.5,1.0,1.5)$ and $\tilde{x}_2 = (1.5,2.0,2.5)$ be the two initial root of the equation. Then

$\tilde{x}_1^* = (1.0,0.5,0.5)$ and $\tilde{x}_2^* = (2.0,0.5,0.5)$ by definition 3.1.

Then from the equation (5.1), we have,

$$\tilde{f}(\tilde{x}_1^*) = (-5.0,0.5,0.5)$$

$$\tilde{f}(\tilde{x}_2^*) = (14.0,0.5,0.5)$$

Now, $\Re(\tilde{f}(\tilde{x}_1^*)) < 0$ and $\Re(\tilde{f}(\tilde{x}_2^*)) > 0$.

Then the root lies between $\tilde{x}_1^*$ and $\tilde{x}_2^*$.

Hence, $\tilde{x}_3^* = \dfrac{\tilde{x}_1^* + \tilde{x}_2^*}{2} = (1.5,0.5,0.5)$ and corresponding root is $\tilde{x}_3 = (1.0,1.5,2.0)$ by definition 3.1

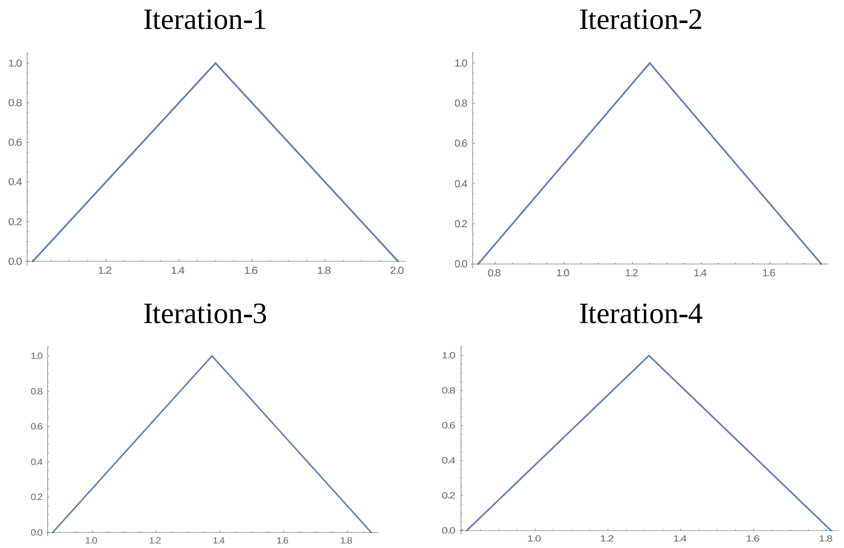
Where, $\Re(\tilde{f}(\tilde{x}_3^*)) > 0.$ Then the root lies between $\tilde{x}_1^*$ and $\tilde{x}_3^*$.

Proceeding in this way, we have the following table.

Table 1: Roots of the fuzzy non-linear equation.

S/N	Root lies between	$\tilde{x}_n^*$	Roots $\tilde{x}_n$	Sign of $\Re(\tilde{f}(\tilde{x}_n^*))$
1	$\tilde{x}_1^*$ and $\tilde{x}_2^*$	$\tilde{x}_3^* = (1.5,0.5,0.5)$	$\tilde{x}_3 = (1.0,1.5,2.0)$	
2	$\tilde{x}_1^*$ and $\tilde{x}_3^*$	$\tilde{x}_4^* = (1.25,0.5,0.5)$	$\tilde{x}_4 = (0.75,1.25,1.75)$	(-)ve
3	$\tilde{x}_4^*$ and $\tilde{x}_3$	$\tilde{x}_5^* = (1.375,0.5,0.5)$	$\tilde{x}_5 = (0.875,1.375,1.875)$	(+)ve
4	$\tilde{x}_4^*$ and $\tilde{x}_5$	$\tilde{x}_6^* = (1.3125,0.5,0.5)$	$\tilde{x}_6 = (0.8125,1.3125,1.8125)$	(-)ve
5	$\tilde{x}_6$ and $\tilde{x}_5$	$\tilde{x}_7^* = (1.34375,0.5,0.5)$	$\tilde{x}_7 = (0.84375,1.34375,1.84375)$	(-)ve
6	$\tilde{x}_7^*$ and $\tilde{x}_5$	$\tilde{x}_8^* = (1.359375,0.5,0.5)$	$\tilde{x}_8 = (0.859375,1.359375,1.859375)$	(-)ve
7	$\tilde{x}_8^*$ and $\tilde{x}_5$	$\tilde{x}_9^* = (1.3671875,0.5,0.5)$	$\tilde{x}_9 = (0.8671875,1.3671875,1.8671875)$	(+)ve
8	$\tilde{x}_8^*$ and $\tilde{x}_9^*$	$\tilde{x}_{10}^* = (1.36328125,0.5,0.5)$	$\tilde{x}_{10} = (0.86328125,1.36328125,1.86328125)$	(-)ve

Hence the required root is $\tilde{x}_{10} = (0.86328125,1.36328125,1.86328125)$ correct up to two decimal places. The graphical representations of the sequence of approximate roots are shown in fig. 3 and the optimum solution of fuzzy non-linear equation is shown in fig. 4.



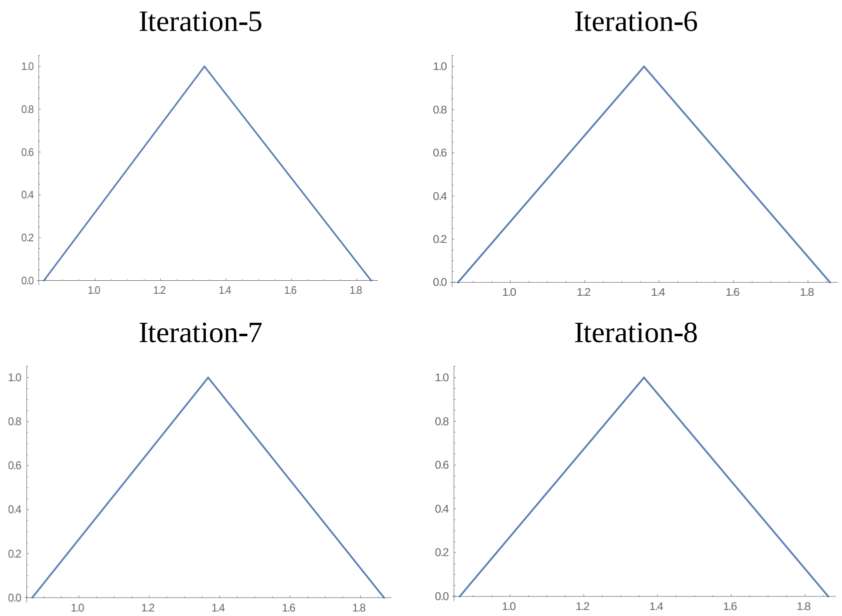


Figure 3: Graphical representation of approximate solutions of the given fuzzy nonlinear equation.

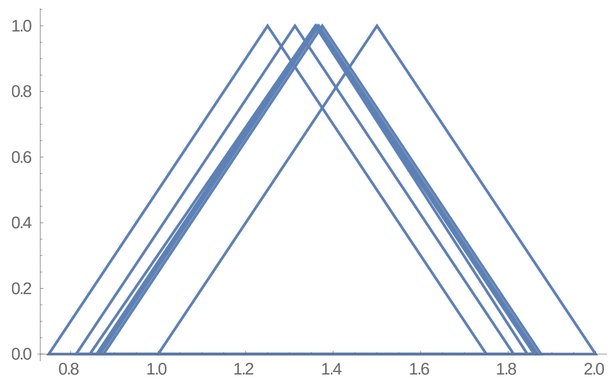


Figure 4: Graphical representation of optimum solution of a fuzzy non-linear equation given in the above example.

6. CONCLUSIONS

In this paper, we have defined new arithmetic operations for triangular fuzzy number. Then, using these operation, modified bisection method is proposed for solving fuzzy non-linear equations. This modified method seems to be very easy to employ with reliable results.

REFERENCES

[1] Subash R, Sathya S, (2011) Numerical solution of fuzzy modified Newton-Raphson method for solving nonlinear equations, International Journal of Current Research, 3(11), pp. 390-392.

[2] Saha G K, Shirin S, (2012) A new approach to solve fuzzy non-linear equations using fixed point iteration algorithm, J. Bangladesh Math. Soc., (32), pp.15-21.

[3] Saha G K, Shirin S, (2013) Solution of Fuzzy Non-linear Equation using Bisection algorithm, Dhaka Univ. J. Sci. 61(1), pp. 53-58.

[4] Saha G K, Shirin S, (2015) Fuzzy Version of Secant Method to Solve Fuzzy Non-linear Equations, GANIT J. Bangladesh Math. Soc., 35, pp. 127-134.

[5] Abbasbandy S, Asady B, (2004), Newton's method for solving fuzzy nonlinear equations, Appl. Math.Comput., 159, pp. 349-356.

[6] Ali M Y, Chowdhury K R, Sultana A, Khan A F M K, (2016) Solution of Fuzzy Non-linear Equations over Triangular Fuzzy Number using Modified Secant Algorithm, Annals of Pure and Applied Mathematics, 12(1), pp. 41-47.

[7] Klir G J, Yuan B, (1997) *Fuzzy sets and fuzzy logic Theory and Applications*, Prentice-Hall of India Private Limited, New Delhi, pp.1-117.

[8] Ming Ma, Menahem Friedman, Abraham Kandel, (1999) A new fuzzy arithmetic, Fuzzy Sets and Systems 108, pp. 83-90.