Comparative Study among Different Delay Estimation Models at Signalized Intersection

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Abstract: ***Intersection delay is a performance measure criteria for signalized intersection. Several models have been developed to estimate delays at intersection. This paper presents study of traffic at an arterial road section of Route 18 in New Brunswick, New Jersey, USA which was modeled in Paramics. A similar model was constructed by Synchro to validate simulation model and optimize signals. Simulation data were collected using detectors from ten signalized intersections under three different demand levels. Arrival times collected through the detectors. Average delays were estimated for signalized intersections using MATLAB code. Delay estimated by Webster's equation perform poorly. Modified Webster's equation predict intersection delay better for the downstream intersections of the arterials. Under high through traffic, Newell's equation is more accurate than others.***

Keywords: Intersection delay, Paramics, Webster's equation, Newell's equation, Synchro, Demand.

1. Introduction

Vehicular delay is an important tool for assessing the operation of signalized intersections. The average delay at signalized intersection is usually used for both design and performance evaluation procedures. It is also used to estimate exhaust emission, noise, and fuel consumption. Delay at a signalized intersection is associated with the time lost by a vehicle because of the operation of the signal. It is a complex variable sensitive to various parameters such as signal setting, traffic characteristics, driver's behavior etc. Numbers of mathematical formula

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are developed by Webster (1968), Miller (1968), Newell (1965), Van As (1990), and Fambro et al. (1997) to estimate traffic delays at signalized intersections.

A section of Route 18 in New Brunswick road network modeled in Paramics is used for this research project. To observe arrival patterns at different demand levels, three different origin-destination (OD) matrices were considered. All signals were subsequently optimized using Synchro for the same traffic demands used in the Paramics model. In this study, the simulation time period was to be taken ten hours and numbers of simulation required for each demand level were determined based on statistical significance.

Webster’s, Modified Webster’s, and Newell’s equations were employed for comparative study using simulation as the basic evaluation tool. Traffic arrivals of signalized intersections are studied in order to study the fitness of these arrivals to a given probability distribution such as uniform, Poisson, binomial and negative binomial. In many situations, standard arrival models could not predict arrival pattern adequately. Therefore, a time series model is introduced to propose a better solution to this problem.

2. Literature Review

Miller (1963), Allsop (1972), Hutchinson (1972) and many other researchers used stochastic equilibrium as delay models at a signalized intersection which consists of three delay components - deterministic delay, random delay, and delay due to within-cycle randomness of arrivals. The basis of deterministic delay is traffic flow theory. It assumes that arrival and departure of vehicles are continuous variable described by flow rates in the time-space domain. Stochastic component is based on traffic demand and service time distribution described by the steady state queuing theory. Deterministic delay is associated with red signals and random delay is mainly caused by over flow queue. Third delay component deals with within-cycle randomness of arrivals are negligible and are not studied extensively.

Allsop [1] worked on vehicular acceleration and deceleration delay component of an intersection. Rouphail et al. [2] researched on uniform arrival and departure rate and its effect on delay estimation. Viti [3] focused on estimation of delay due to over saturated traffic with initial queue. He also considered stochastic fluctuation of arrival. Cronje [4] tried to relate queue length with intersection delay and draw comparative study among different delay calculation techniques. When arrival flow is close to capacity, i.e. degree of saturation is between 1 and 1.1, deterministic and stochastic models cannot predict delay. A study conducted by Akcelik [5] showed that time-dependent delay models can fill the gaps between two types of models and give more realistic results in predicting delay at signalized intersections.

3. Delay models

Prior to describing different delay prediction models, notations and terminologies related to the equations are given below:

Assume,

d = average delay (sec /vehicle)

c = cycle length (sec)

g = effective green time (sec)

$\lambda = \frac{g}{c}$ = proportion of cycle that is effective green (1)

s = saturation flow (vehicle/sec)

Q_0 = average overflow at end of cycle

q = average number of arrival per unit time

x = degree of saturation = $\frac{\text{average number of arrivals per cycle}}{\text{maximum number of departures per cycle}}$ (2)

$I = \frac{\text{variance of the number of arrivals per cycle}}{\text{mean number of arrivals per cycle}}$ (3)

$y = \frac{q}{s} = \frac{\text{average arrival rate}}{\text{saturation flow}}$ (4)

μ = a dimensionless measure of spare capacity of the approach $= \frac{(sg - qc)}{\sqrt{Isg}}$ (5)

H (μ) is a function obtained by numerical integration [6].

$H(\mu) = \exp[-\mu - (\mu 2/2)]$; where $\mu = (1-x)(sg)^{0.5}$ (6)

Some of the analytical delay models described in the following section:

A. Webster's equation

Webster's delay equation based on Poisson arrivals is the first and most widely used model. The first term of equation is for average delay when arrivals are regular and traffic flow is considered as analogous to fluid flow. The second term represents random or stochastic delay estimate delay due to overflow and the third term is for the adjustment of overestimation of delays.

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{1/3} x^{2+5\lambda} \tag{7}$$

B. Modified Webster's equation

Webster's simple model does not incorporate variability of arrival to encounter upstream signal effect on arrival process completely. However, adding variability index I with random delay component, Webster's model is found to address this problem. The equation is shown below is known as extended model of Webster's or Modified Webster's model [6]

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{I\,x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{1/3} x^{2+5\lambda} \tag{8}$$

$$\text{Where, } I = \frac{\text{Variance of arrival per cycle}}{\text{Mean of arrival per cycle}} \tag{9}$$

C. Newell's equation

Newell considered traffic as a fluid flowing at a random variable arrival rate with a mean value of "q" and flowing out at a fixed rate "s" during effective green time as long as any accumulated demand remains. Newell's expression is:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{E\,(Q_0)}{q} + \frac{I(1-\lambda)}{2s\,(1-\lambda x)^2} \tag{10}$$

$$\text{Where expected queue, } E\,(Q_0) = H(\mu).\frac{I_a\,x}{2\,(1-x)} \tag{11}$$

$$\text{So, } d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{IH(\mu)x}{2q(1-x)} + \frac{I(1-\lambda)}{2s\,(1-\lambda x)^2} \tag{12}$$

Function H (μ) reflects magnitude of filtering. It is also known as degree of filtering of traffic flow by a signal. $H(\mu) = \exp(-\mu - \mu^2/2)$

and $\mu = (1-x)\ C^{1/2}$. H is a decreasing function obtained by numerical integration [7].

4. Research Methods

A section of Route 18 in New Brunswick road network modeled in Paramics was used for this research project. Signalized operations of a portion of Route 18 modeled in Paramics were adjusted for each demand levels according to the optimized signal timing. This timing was obtained from the corresponding Synchro model. Each demand level was simulated using different seed values. Arrival times of vehicles were recorded by detectors in Paramics. Then variability index and average delay for each signal were calculated using MATLAB code developed for this research. Subsequently calculating variability index values, average delay of each signal was also estimated using the output of corresponding simulation runs. Delay of a vehicle at a signal is the difference between actual travel time and free flow travel time of the vehicle. Then, average vehicular delay for each signal was calculated using a MATLAB. In this study, the simulation time period was to be taken ten hours and numbers of simulation required for each demand level were determined based on statistical significance. Webster's, Modified Webster's, and Newell's equations were employed for comparative study using simulation as the basic evaluation tool.

5. Model Description

A section of Route 18 in New Brunswick, New Jersey shown in Figure 1(a) was modeled using Paramics simulation tool. The starting point of the network is located approximately near the intersection of Route 18 (Memorial Parkway) and Route 27. The end point of the network is located at the intersection of County Road 516 (Old Bridge Matawan Road) and County Road 687 in Old Bridge. The model has seventy six trip generating zones and eleven signalized intersections. An approximate equivalent model of a section of Route 18 shown in Figure 1(b) was also built in Synchro to optimize signal timings for three different demand levels. Snapshot of some of the intersections modeled in Paramics are presented in the Figure 2.

Figure 1(a): *Snapshot of a section of road network modeled in Paramics for Route 18, New Brunswick, New Jersey*

Loop detectors were placed into the Paramics model to collect traffic data. Three detectors were placed for each signalized intersection.

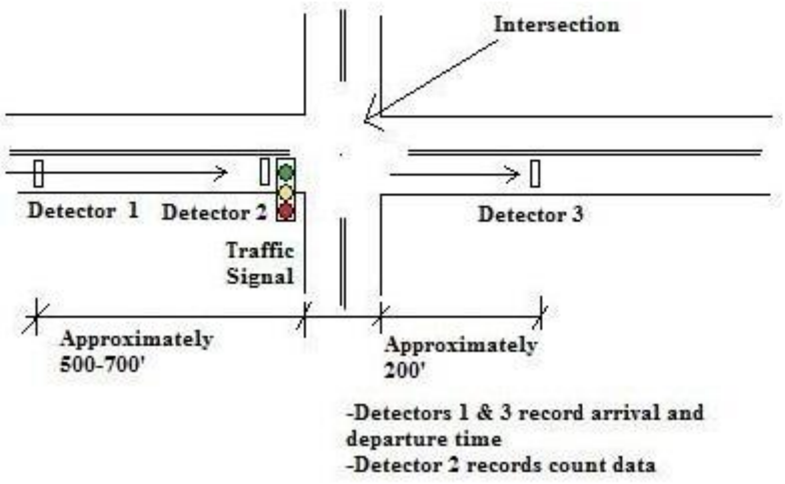

Figure 3: Position of detectors at an intersection

To study traffic arrival patterns under different demand levels, three demand levels were used for the simulation and evaluation. They were numbered as one to three ranging from low to high based on volume to capacity (v/c) ratio.

Table 1: Different demand levels with through traffic

Demand Level	v/c ratio
Demand 1 (Low through traffic)	0.51
Demand 2 (Medium high through traffic)	0.64
Demand 3 (High through traffic)	0.83

6. Model validation

As mentioned earlier, an approximate equivalent to the section of Route 18 in Paramics model was built in Synchro. Therefore, a comparative study was required to conduct between two models. Average delays obtained from signalized intersections were used as the measuring criteria for comparison in this study. Deviation of average intersection delays in Paramics and Synchro models were relatively small (approximately 5% to 12%).

7. Data analysis

Average delays of first ten signalized intersections in the portion Route 18 model are estimated using simulation. Webster's, Modified Webster's, and Newell's delay equations are then employed to compare their estimations with corresponding simulated values.

Table 2: Total average delay (sec/vehicle) at signalized intersections

Traffic condition	Webster	Modified Webster	Newell	Paramics
Low through traffic demand	123.94	148.27	129.32	138.63
Medium through traffic demand	139.56	153.22	141.93	148.19
High through traffic demand	120.95	166.75	131.63	148.47

Under low through traffic, total average delays estimated by Webster's and Newell's equations are 6.95% and 6.72% lower than that delay estimated by the simulation model respectively. On the other hand, total average delay estimated by Modified Webster's equation is 10.6% higher than the simulation based average delay. Under medium through traffic, Webster's and Newell's equations provide 5.82% and 4.22% higher estimates compared with simulation respectively. However, delays based on Modified Webster's equation are 3.39% lower than the simulation based results. Under high through traffic demand, total average delays estimated by Webster's and Newell's equations are 18.53% and 12.31% lower than the simulated delays respectively. On the other hand, total average delay estimated using Modified Webster's equation is 11.34% higher than the delays estimated based on simulation.

Based on Figure 4(a), slope of Modified Webster's equation is 1.09, which represents over estimation compared with Paramics estimates. Webster's equation has a slope of 0.89, which depicts an under estimation of delays. From Figure 4(b), Newell's model has a slope of 0.94, which is better than Webster's equation however still lower than simulation.

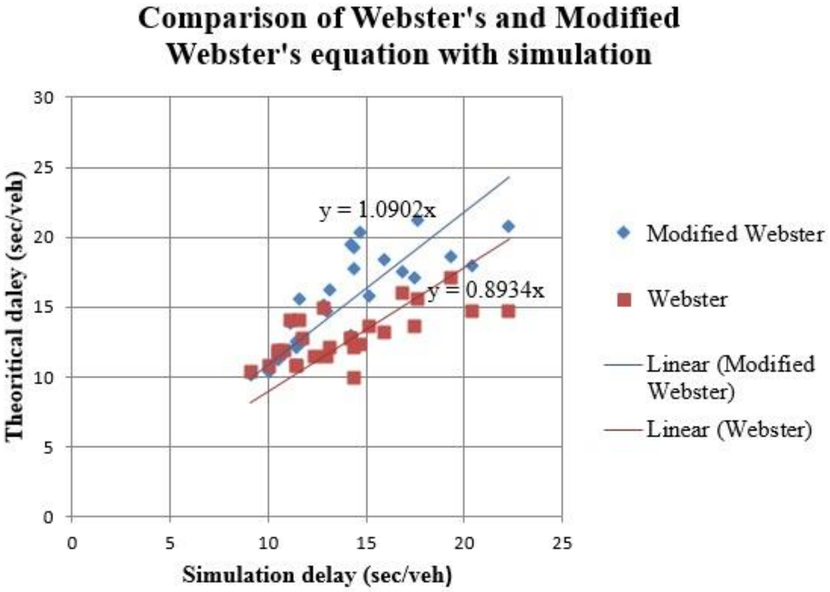

Figure 4 (a) : Average delays from Webster's, Modified Webster's equations and Simulation

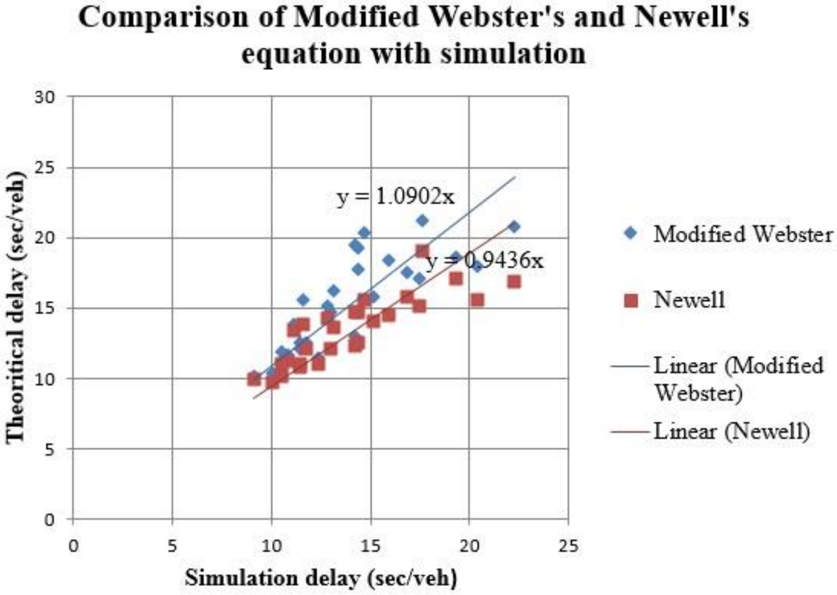

Figure 4 (b) : Average delays from Modified Webster's, Newell's equations and Simulation

Under low traffic, Newell’s equation performs relatively better than Modified Webster’s equation for the first eight signals. However, Modified Webster’s model is best for the last two signalized intersections. Newell’s equation performs relatively better than other models for the first and third signals under medium traffic. Modified Webster’s equation in turn performs better for the second, and fourth to seventh signals. On the other hand, Modified Webster’s model has a better prediction performance for the last three signals. Under high traffic, Newell’s equation performs better than the other two models for the first eight intersections. Modified Webster’s model performs better for the last two signalized intersections.

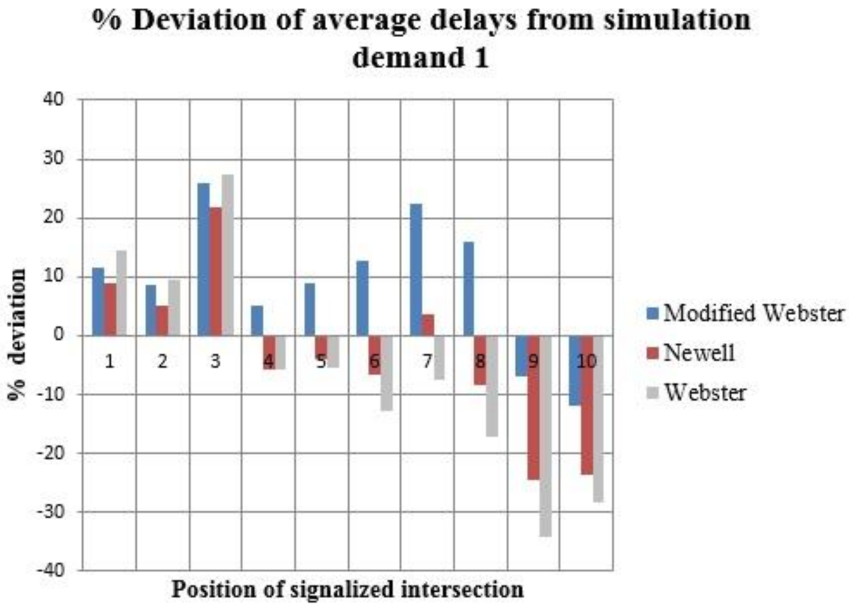

Figure 5(a): Percent deviation from the simulated average delays under low through traffic

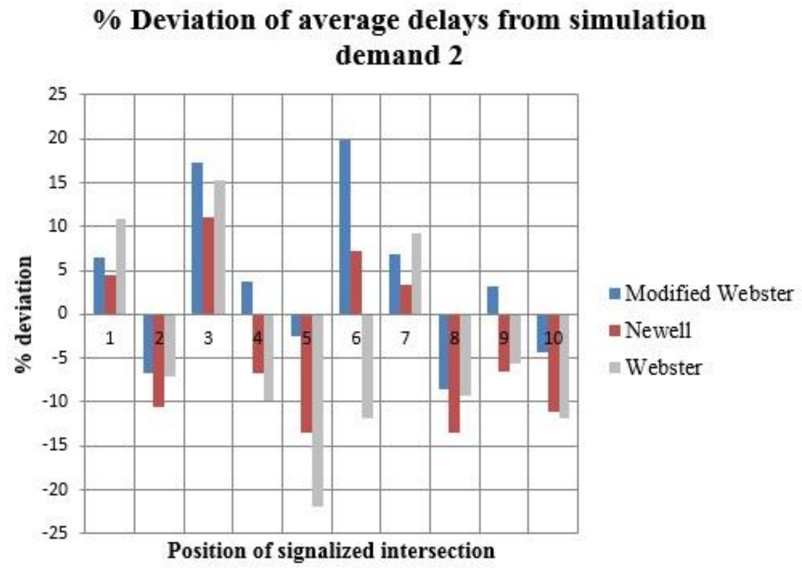

Figure 5(b): Percent deviation from the simulated average delays under medium through traffic

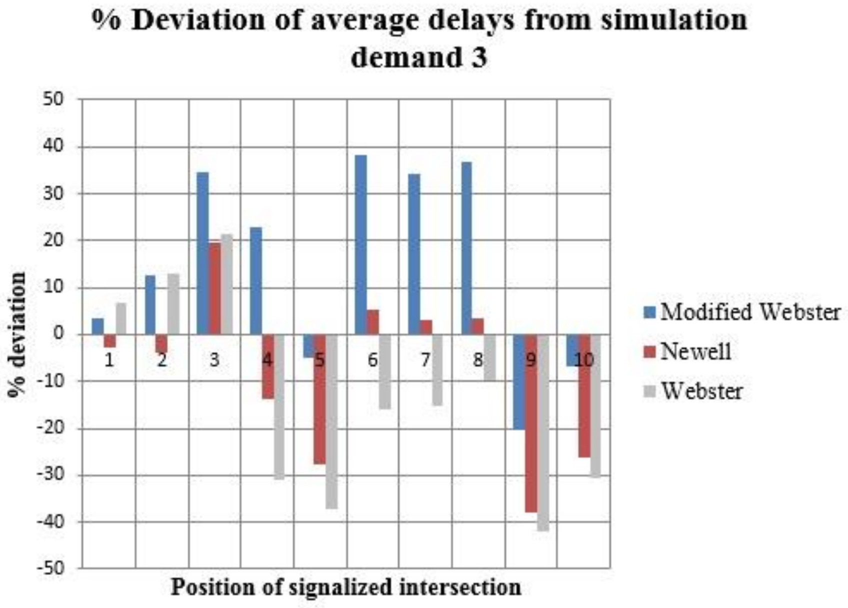

Figure 5(c): Percent deviation from the simulated average delays under high through traffic

8. Conclusion

From the simulations performed in this research, it can be concluded that, traffic characteristics of multilane arterial system are different than single lane. Vehicle lane changing, speed variation, and building of horizontal queue at intersection are observed in a multilane road. Newell's equation performs better than Webster's and Modified Webster's equations for delays estimation at signalized intersection. Delays obtained from Newell's and Webster's equations are slightly lower than the simulation results. On the other hand, Modified Webster's equation estimates higher average delays than simulation. Webster's equation cannot predict average delay adequately.

Deviation among average delays obtained from simulation and theoretical delay equations increase with the increase of variability index. Modified Webster's and Newell's equations produce significant deviation from delay estimated by simulation at high variability index. Webster's equation cannot predict average delay adequately. Binomial and Poisson arrival distributions are observed at vehicle arrivals of upstream signals. On the other hand, arrivals of downstream signals are negative binomial distribution. In many cases, arrival count data do not fit with any standard distribution. Traffic arrivals of the arterial road are influenced by the presence of closely spaced signalized intersections. Moreover, turning movements at the upstream signals, presence of un-signalized mid blocks, and travel time variation contribute to the arrivals of downstream signals. Stochastic delays of a signalized intersection are depends on traffic arrivals. Therefore, predictions of traffic arrival are needed for accurate estimation of vehicular delays at signalized intersections.

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